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AVERAGE AND PROBABILITY.

Conducted by B.F.FINKEL, Kidder, Missouri. All contributions to this department should be sent to him.

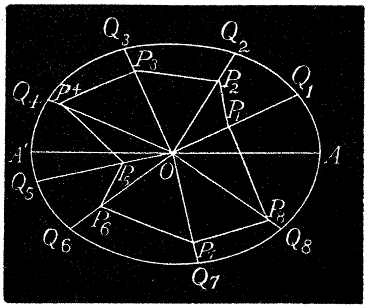
SOLUTIONS TO PROBLEMS.

4. Proposed by G. B. M. ZERR, A. M., Principal of High School, Staunton, Virginia.

Four points taken at random in each half, made by the transverse axis, of an ellipse, are joined in such a way by straight lines as to enclose an octagonal surface; find the mean area of this surface.

II. Solution by F. P. MATZ, M. Sc., Ph. D., Editor of the Department of Mathematics in the "New England Journal of Education", and Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.

Let P_1, P_2, ..., P_8 represent the eight random points; OQ_1, OQ_2, ..., OQ_8 the radius vectors drawn through these points; $x_1=OP_1$, $x_2=OP_2$ ...., $x_8=OP_8$ the distances of the random points from the center of the ellipse. The random points may range over the radius vectors on which they lie. The point Q_1 may range over the elliptic arc AQ_2; that is, the number of radius vectors on which P_1 may lie is proportional to the length of the elliptic arc AQ_2. The points Q_2, Q_3, ..., Q_8 may range respectively over the elliptic arcs AQ_3, AQ_4, AA', $A'Q_6$, $A'Q_7$, $A'Q_8$, $A'A$. Represent the polar co-ordinates of the point Q_1, Q_2, ..., Q_8 by (r_1, θ_1), (r_2, θ_2) ...., (r_8, θ_8); then area of the octagonal surface $P_1P_2 \ldots P_8P_1=A=$ the sum of the area of the eight triangles P_1OP_2, P_2OP_3, ..., P_8OP_1; that is, $A=\frac{1}{2}[x_1x_2 \sin(\theta_2-\theta_1)+x_2x_3 \sin(\theta_3-\theta_2)+ \ldots + x_8x_1 \sin(\theta_1-\theta_8)]$.



Representing the specified *elliptic* arcs by l_1, l_2, ..., l_8, the required mean area becomes

$$A=\frac{1}{\triangle}\int_0^{l_8}\int_0^{r_8}\int_0^{l_7}\int_0^{r_7}\int_0^{l_6}\int_0^{r_6}\int_0^{l_5}\int_0^{r_5}\int_0^{l_4}\int_0^{r_4}\int_0^{l_3}\int_0^{r_3}\int_0^{l_2}\int_0^{r_2}\int_0^{l_1}\int_0^{r_1} A\, ds_8dx_8 \times$$

$ds_7dx_7ds_6dx_6ds_5dx_5ds_4dx_4ds_3dx_3ds_2dx_2ds_1dx_1$ .... (1), in which

$$\triangle=\int_0^{l_8}\int_0^{r_8}\int_0^{l_7}\int_0^{r_7}\int_0^{l_6}\int_0^{r_6}\int_0^{l_5}\int_0^{r_5}\int_0^{l_4}\int_0^{r_4}\int_0^{l_3}\int_0^{r_3}\int_0^{l_2}\int_0^{r_2}\int_0^{l_1}\int_0^{r_1} ds_8dx_8 \times$$

$ds_7dx_7ds_6dx_6ds_5dx_5ds_4dx_4ds_3dx_3ds_2dx_2ds_1dx_1$.

From the ellipse, as per *Conic Sections*, r^2_1, r^2_2, ..., $r^2_8=$

$\frac{b^2}{1-e^2\cos^2\theta_1}$, $\frac{b^2}{1-e^2\cos^2\theta_2}$, ..., $\frac{b^2}{1-e^2\cos^2\theta_8}$; and the superior integral limits of

$x_1, x_2, \dots, x_8$, as obtained from this system of equations.

Differentiating these equations, we have respectively,

$$\left(\frac{dr_1}{d\theta_1}\right)^2, \left(\frac{dr_2}{d\theta_2}\right)^2, \dots, \left(\frac{dr_8}{d\theta_8}\right)^2 = \frac{b^2e^4\sin^2\theta_1\cos^2\theta_1}{(1-e^2\cos^2\theta_1)^3}, \frac{b^2e^4\sin^2\theta_2\cos^2\theta_2}{(1-e^2\cos^2\theta_2)^3},$$

$$\dots, \frac{b^2e^4\sin^2\theta_8\cos^2\theta_8}{(1-e^2\cos^2\theta_8)^3} \dots (2).$$

By means of the formula for the rectification of plane curves represented by polar co-ordinates, we have from (2)

$$\int_0^{l_1} ds_1 = b\int_0^{\theta_1} \frac{\sqrt{[1-e^2(2-e^2)\cos^2\theta_1]}}{(1-e^2\cos^2\theta_1)^{\frac{3}{2}}}\,d\theta_1;$$

$$\int_0^{l_2} ds_2 = b\int_0^{\theta_2} \frac{\sqrt{[1-e^2(2-e^2)\cos^2\theta_2]}}{(1-e^2\cos^2\theta_2)^{\frac{3}{2}}}\,d\theta_2;$$

$$\int_0^{l_3} ds_3 = b\int_0^{\theta_3} \frac{\sqrt{[1-e^2(2-e^2)\cos^2\theta_3]}}{(1-e^2\cos^2\theta_3)^{\frac{3}{2}}}\,d\theta_3;$$

$$\int_0^{l_4} ds_4 = b\int_0^{\pi} \frac{\sqrt{[1-e^2(2-e^2)\cos^2\theta_4]}}{(1-e^2\cos^2\theta_4)^{\frac{3}{2}}}\,d\theta_4;$$

$$\int_0^{l_5} ds_5 = b\int_0^{\theta_5} \frac{\sqrt{[1-e^2(2-e^2)\cos^2\theta_5]}}{(1-e^2\cos^2\theta_5)^{\frac{3}{2}}}\,d\theta_5;$$

$$\int_0^{l_6} ds_6 = b\int_{\pi}^{\theta_6} \frac{\sqrt{[1-e^2(2-e^2)\cos^2\theta_6]}}{(1-e^2\cos^2\theta_6)^{\frac{3}{2}}}\,d\theta_6;$$

$$\int_0^{l_7} ds_7 = b\int_{\pi}^{\theta_7} \frac{\sqrt{[1-e^2(2-e^2)\cos^2\theta_7]}}{(1-e^2\cos^2\theta_7)^{\frac{3}{2}}}\,d\theta_7;$$

$$\int_0^{l_8} ds_8 = b\int_{\pi}^{2\pi} \frac{\sqrt{[1-e^2(2-e^2)\cos^2\theta_8]}}{(1-e^2\cos^2\theta_8)^{\frac{3}{2}}}\,d\theta_8.$$

The evaluation of the thirty-two integrals indicated in (1) is a labor sufficient to discourage even a mathematical Hercules.

6. Proposed by J. F. W. SCHEFFER, A. M., Hagerstown, Maryland.

Find the average length of all the diameters that can be drawn in a given ellipse.

Solution by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.

Let $2r$ represent any diameter; then from the *central-polar* equation of the ellipse, $r^2 = \dfrac{b^2}{1-e^2\cos^2\theta}$, we have $2r = \dfrac{2b}{\sqrt{(1-e^2\cos^2\theta)}}$,

$$\frac{dr}{d\theta} = \frac{be^2\sin\theta\cos\theta}{(1-e^2\cos^2\theta)^{\frac{3}{2}}};\ \text{and}\ \frac{ds}{d\theta} = b\sqrt{\left(\frac{1-e^2(2-e^2)\cos^2\theta}{(1-e^2\cos^2\theta)^3}\right)}.$$

Since the number of diameters that can be drawn in an elliptic quadrant is proportional to the length of the elliptic arc bounding that quadrant, the required average length becomes